# Mental Math Tricks and More

**Daryl Stephens, ETSU**

Students in grades 5–12 in Texas compete in a contest each spring known as Number Sense. It's governed by the University Interscholastic League (UIL) based at the University of Texas (Austin).[1] The test itself takes 10 minutes and usually has 80 questions. Students are given the test and a pencil with the eraser taped over. Problems must be answered using only mental calculations — no calculators, scratch work, or erasures. Every tenth problem is starred and requires only an answer within 5% of the exact answer. Tests are scored by taking five times the number right minus four times the number wrong up through the last problem done. The ones left blank at the end aren't counted, but any problems skipped before the last one worked are counted wrong.

For test problem sequencing and sample tests, see other handouts.

## Sample Shortcuts (in no particular order)

I. **Using algebraic identities**

A. $5 \times 17 + 5 \times 3 =$ _______ Use the **distributive property** to turn it into $5 \times 20 = \boxed{100}$ .

B. $12^2 - 8^2 =$ ________ Use the **difference of squares** to think of $(12 + 8)(12 - 8) = (20)(4) = \boxed{80}$ .

II. **Multiplication shortcuts**

A. **Multiply by powers of 10**. These tricks are usually taught in regular math classes at the middle school level and involve annexing the right number of zeros or shifting a decimal point. They might possibly need teaching to fourth, fifth or sixth graders.

B. **Multiply by 50**. Multiply by 100 and divide by 2 (or vice-versa). Or multiply by 5 and affix a 0 – whichever is easier.

C. **Multiply by 25**. Multiply by 100 and divide by 4 (or vice-versa).

D. **Multiply by 75**. Multiply by 100, divide by 4, and multiply by 3.

E. **Multiply by 33⅓**. Multiply by 100 and divide by 3.

---

[1] UIL governs a number of other academic contests in Texas public schools, such as calculator, drama, speech, debate, keyboarding, journalism, one-act play, computer science, reading maps/charts/graphs, spelling, ready writing, picture memory, and storytelling. It also governs middle and high school sports. Similar contests are sponsored by the Private Schools Interscholastic Association.

F. **Squaring a two-digit number ending in 5**. The product ends in 25. Then ly the tens digit by 1 higher and write that product in front. Example: $75^2 = 5625$. (Write down 25, then multiply 7 × 8.)

G. **Multiply two 2-digit numbers in the same decade whose units digits add to 10**. (E.g. 24 × 26, 87 × 83). Multiply the units digits and write that down; if the product is less than 10, make it a two-digit number by writing a 0 in front; e.g. 1 × 9 would be written as 09. Then multiply the tens digit by one higher and write that product down in front. Example:

1. 87 × 83 = ___21 (7 × 3 = 21)
2. 87 × 83 = $\boxed{7221}$ (8 × 9 = 72)

H. **2-digit × 2-digit in general**. Use FOIL in reverse from algebra. Carry digits in your head. Example: 42 × 37 = 1554.

1. 42 × 37 = ___4 2 × 7 = 14; write down the 4 and carry the 1.
2. 42 × 37 = ___54 4 × 7 + 2 × 3 + 1 (carried from the last step) = 35. Write down the 5 and carry the 3 in your head.
3. 42 × 37 = $\boxed{1554}$ 4 × 3 + the carried 3 = 15.

I. **Multiply by 11**. Start with writing down the ones digit of the other factor. Then add each digit to its neighbor to the right, carrying if necessary. Then write down the first digit (plus anything that is carried mentally). Example: 11 × 6823.

1. 11 × 6823 = ______3 (Write down the 3.)
2. 11 × 6823 = _____53 (2 + 3 = 5)
3. 11 × 6823 = ____053 (8 + 2 = 10. Carry the 1 in your head.)
4. 11 × 6823 = ___5053 (6 + 8 + 1[carried] = 15. Carry the 1 again.)
5. 11 × 6823 = $\boxed{75{,}053}$ (The first digit is 6, plus the 1 we carried.)

J. **Multiply by 12**. This is sometimes referred to as "double the digit and add to its neighbor on the right." Example: 57 × 12 = 684

1. 57 × 12 = ___4 Think 7 × 2 = 14; carry the 1.
2. 57 × 12 = ___84 Think 5 × 2 = 10; add 7 so 10 + 7 = 17; add the carried 1 to get 18. Write 8 and carry the 1.
3. 57 × 12 = $\boxed{684}$ Take the 5, then add the carried 1; write 6.

K. **Using Difference of Squares to help in multiplying**. We can use the algebra fact $(a+b)(a-b) = a^2 - b^2$ to multiply two numbers that are an equal distance from each other. Example: 36 × 44. Both 36 and 44 are 4 units away from 40, so think of this as (40 + 4)(40 – 4) and work it as $40^2 - 4^2 = 1600 - 16 = \boxed{1584}$ .

L. **Multiply by 125.** Multiply by 500 and divide by 4. So, for example, 48 × 125 would be $\mathbf{48} \times \frac{\mathbf{500}}{\mathbf{4}}$ = 12 × 500 = $\boxed{6000}$ .

M. **Double and Half.** For some multiplication problems (see below), it may be easier to multiply twice one factor by half the other. This is especially good for

1. An even number times a multiple of 5 (e.g. $35 \times 18 = 70 \times 9$)
2. A number times a multiple of 11 (e.g. $22 \times 13 = 11 \times 26$)
3. One factor that is a mixed number ending in ½. ($7\frac{1}{2} \times 16 = 15 \times 8$)

N. **Multiplying by 9.** Multiply the other number by 10, then subtract it. (In other words, $9n = 10n - n$.) Examples:

$9 \times 23 = 230 - 23 = \boxed{207}$

$19 \times 42 = 20 \times 42 - 42 = 840 - 42 = \boxed{738}$

III. **Sum of a series**

A. Example: $4 + 6 + 8 + 10 + 12 =$ _______
Add numbers in pairs from opposite ends inwards:
$(4 + 12) + (6 + 10) + 8 = 16 + 16 + 8 = \boxed{40}$

B. Example: $1 + 2 + 3 + 4 + 5 + 6 + 7 =$ _________
Use the formula that $1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$, where $n$ is the last term (and the number of terms). So the sum is $7 \times 8 \div 2 = \boxed{28}$.

C. In an arithmetic sequence not starting with 1 and increasing by 1, use the formula $S_n = (\text{first} + \text{last}) \cdot \frac{n}{2}$, where $n$ is the number of terms.

Example: $10 + 12 + 14 + 16 + 18 + 20 + 22 + 24 =$ ____________
$10 + 24 = 34$; there are 8 terms; $8 \div 2 = 4$; so $34 \times 4 = \boxed{136}$.

IV. **Number Theory and Related Topics**

A. Useful fact: $\text{GCF}(a, b) \times \text{LCM}(a, b) = a \times b$. (Find the GCF and LCM by whatever method seems easiest to do mentally.)

B. How many positive integer divisors does a whole number have?

1. Find the prime factorization of the number (use exponents).
2. Add 1 to each exponent.
3. Multiply the new exponents together; this gives the number of positive integer factors.

Example: How many positive integer divisors does 40 have?
$40 = 2^3 \cdot 5^1$. The number of divisors, then, is $4 \times 2 = \boxed{8}$.

C. Note on adding and subtracting fractions: For mental arithmetic, it may often be easier to use $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$ $\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$ (or write in lowest terms rather than changing to common denominators and computing.

D. Writing a repeating decimal as a fraction:

1. If a repeating decimal has all digits after the decimal as repetends, write a fraction with the repetend as the numerator and a denominator with the same number of 9s as the number of repeating digits. Then write in lowest terms. So $0.\overline{45} = \frac{45}{99} = \frac{5}{11}$
2. Repeating decimals such as $0.5\overline{8}$ :
   a. Take 58 – 5 = 53
   b. Divide by 90 (and simplify if possible): $\frac{53}{90}$

   Another example: $0.2\overline{7} = \frac{25}{90} = \frac{5}{18}$

   (Think of $.a\overline{b} = \frac{ab - a}{90}$ where $a$ and $b$ are digits.)

V. **Sets**

A set with $n$ elements has $2^n$ subsets.

VI. **Harder or More Obscure Multiplication Tricks**

A. **Multiply two 3-digit numbers**. Let's write it as $abc \times def$, where the letter represent digits.

1. Multiply ones ($f \times c$) and write down the ones; carry.
2. Do $(b \times f) + (c \times e)$ + carried digit; write down the ones and carry.
3. Do $(a \times f) + (c \times d) + (b \times e)$ + carried digit; write down 1s and carry.
4. Do $(a \times e) + (b \times d)$ + carried digit; write down ones and carry.
5. Multiply $a \times d$ and add carried digit; write it down.
6. Example: 123 × 456

| | | |
|---|---|---|
| a. | 6 × 3 = 18 | 8 |
| b. | (6 × 2) + (5 × 3) + 1 = 28 | 88 |
| c. | (6 × 1) + (4 × 3) + (5 × 2) + 2 = 30 | 088 |
| d. | (5 × 1) + (4 × 2) + 3 = 16 | 6088 |
| e. | (4 × 1) + 1 = 5 | 56,088 |

B. **Multiply a 2-digit number by 101.** Write the 2-digit number next to itself. Example: 52 × 101 = 5252.

C. **Multiply a 2-digit number by 111**. Example: 111 × 58

| | | | |
|---|---|---|---|
| 1. | Write down the ones digit. | 111 × 58 = 8 | |
| 2. | Add the tens and ones; carry. | 111 × 58 = 38 | (5 + 8 = 13) |
| 3. | Repeat the previous step. | 111 × 58 = 438 | (5 + 8 + 1 = 14) |
| 4. | Write the tens digit (and carry). | 111 × 58 = $\boxed{6438}$ | (5 + 1 = 6) |

D. **Multiply a 3-digit number by 111.** Use right-to-left "sweeps" of 1, 2, 3, 2, and 1 digit. Example: 111 × 234

1. 111 × 234 = ________4 Write down the 4.
2. 111 × 234 = _______74 3 + 4 = 7
3. 111 × 234 = ______974 2 + 3 + 4 = 9
4. 111 × 234 = _____5974 2 + 3 = 5
5. 111 × 234 = $\boxed{25{,}974}$ Write down the 2.

As before, you may have to carry a 1 or 2 in your head.

E. **Multiply a multiple of 7 by 715**. Take the multiple of 7, divide it by 7, then multiply by 5. Write that result as a 3-digit number (padding with 0s if necessary), and then write it again to the left.

Example: 42 × 715. 42 ÷ 7 = 6, 6 × 5 = 30, so the answer is $\boxed{30{,}030}$ .

F. **Multiply a multiple of 7 by 429**. Divide the other number by 7 and multiply by 3. Write the result as a 3-digit number, then repeat the result in front. (This works because $\frac{3}{7} \times \mathbf{1001} = \mathbf{429}$ .) Example: 56 × 429 = ?

56 ÷ 7 × 3 = 24. So the product is $\boxed{24{,}024}$ .

G. **Multiply a multiple of 7 by 572**. Same idea as for 429, but divide by 7 and multiply by 4. So 14 × 572 = $\boxed{8008}$ .

H. **Multiply a multiple of 7 by 858**. Same as for 429, but divide by 7 and multiply by 6. So 21 × 858 = $\boxed{18{,}018}$ .

## VII. Simpler Things

A. **Addition using compensation**. Add a number to one addend and subtract it from the other to simplify the addition. For example, change 67 + 58 into 70 + 55 so that one addend is a multiple of 10.

B. **Subtraction using compensation:** Add (or subtract) the same number to minuend and subtrahend to get easier numbers to subtract. Example: Change 853 – 397 by adding 3 to both numbers and computing 856 – 400. Or turn 502 – 328 into 500 – 326 by subtracting 2 from both numbers.

## VIII. Converting to Other Number Bases

A. Most conversion has no tricks to it; it's just sheer grunt work.

B. **Change base 2 to base 4**. Split the base-2 number into blocks of two digits. Then convert $00_2 = 0_4$; $01_2 = 0_4$; $10_2 = 2_4$; $00_2 = 3_4$. So to answer $1001100_2 = \_\_\_\__4$, partition the base-2 numeral as 1|00|11|00 and rewrite it as $\boxed{1030_4}$ .

Base 4 to 2 would work in a similar fashion. Example: $3102_4 = \boxed{11010010_2}$ .

C. **Change base 2 to base 8**. Similar to changing base 2 to 4, but this time partition the numeral into blocks of 3 digits, remembering this chart:

| Base 2 | 000 | 001 | 010 | 011 | 100 | 101 | 110 | 111 |
|---|---|---|---|---|---|---|---|---|
| Base 8 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |

Examples: $110010011001111_2 = 110\,|\,010\,|\,011\,|\,001\,|\,111_2 = 62317_8$.
$435_8 = 100011101_2$.

## References

Blend, Henrietta. (n.d.) *Oh to Be a Math Wizard!* (Publishing place unknown – picked up at an inservice in Texas in the late 1980s.)

Ray, Doug. (2006). Dr. Numsen's Trick Center. Retrieved September 14, 2007, from http://www.academicmeet.com/tricks/

Skow, Donald P. (1989). *Developing Middle School Number Sense Skills.* Austin, TX: University Interscholastic League, Division of Continuing Education, the University of Texas.

University Interscholastic League. (2007). *EJH Number Sense Problem Sequencing.* Retrieved September 11, 2007, from http://www.uil.utexas.edu/academics/elem_jrhigh/numsense_prob.html

Dr. Daryl Stephens
Mathematics Department
East Tennessee State University
Box 70663
Johnson City, TN 37614-1701
1-423-439-4676
stephen@etsu.edu

## Some Mental Math and Number Sense Test Links

The following pages have lots of tricks on them. The first two are related to the Texas Number Sense Test. The others are of a more general nature.

- http://www.texasmath.org/
- http://www.academicmeet.com/tricks/
- http://www.curiousmath.com/
- http://www.themathlab.com/natural/mental%20math%20tricks/powerzone.htm

Here are some places you can find practice middle school number sense tests:

- http://www.gcisd.net/~luonna.adkins/TMSCA%20Practice%20Tests (Some of the answer keys don't match up correctly with the right tests.)
- http://www.gcisd-k12.org/schools/ctms/clubs_mathsciencenumbersense.htm (Something like 132 tests here! From Cross Timbers Middle School, Grapevine-Colleyville Independent School District)
- http://64.23.52.186/schools/ctms/mathscience/1997/97test02.pdf
- Elementary and Junior High Test Page: http://texasmath.org/DL/EJH/

**High School Tests:**

- http://www.texasmath.org/ – Click on Download on left, then Number Sense tests in main frame

---

### Number Sense Problem Sequencing

(Source: http://www.uil.utexas.edu/academics/elem_jrhigh/numsense_prob.html)

Note: A type of problem from a particular section could appear later in the test.
Example: A base problem could appear as problem #55, but should not appear earlier than problem #21.

### ELEMENTARY

**Problems 1 - 20**

1. Addition, subtraction, multiplication, & division of whole numbers
2. Recognizing place value
3. Rounding off whole numbers
4. Multiplication short-cuts
5. Remainder type problems
6. Even & odd number type problems
7. Expanded notation
8. Sums of whole numbers
9. Roman numerals/arabic numbers

**Problems 21 - 40**

1. Addition/subtraction of fractions with common denominators
2. Addition, subtraction, multiplication, & division of decimal fractions
3. Comparing decimal fractions & common fractions
4. Conversion problems (either way): fraction/decimal, percent/fraction, percent/decimal
5. Order of operations
6. More multiplication short-cuts
7. Ratio/proportion
8. Consumer type problems
9. Problems about prime numbers
10. Greatest common divisor (gcd) & least common multiple (lcm)
11. Conversion problems (either way): length, weight, volume

**Problems 41 - 60**

1. Addition, subtraction, multiplication & division of fractions and mixed numbers

2. Substitution problems
3. Perimeter/area of: square, rectangle, triangle
4. Radius/diameter of a circle
5. Powers & roots of numbers
6. Solving simple equations
7. Sequences
8. Sets
9. Word problems
10. Volume of cube/rectangular box
11. Right triangle problems
12. More multiplication short-cuts
13. Base systems

**Problems 61 - 80**
1. Addition, subtraction, multiplication & division of integers
2. Inverses
3. Basic geometry facts
4. More area problems
5. Squaring two-digit numbers
6. More multiplication short-cuts
7. Powers of numbers
8. More consumer type problems
9. Inequalities
10. Probability
11. More area problems: parallelogram, rhombus, trapezoid
12. Coordinate geometry - number line
13. More percent type problems

## JUNIOR HIGH

**Problems 1 - 20**
1. Addition, subtraction, multiplication & division of whole numbers, fractions, and decimals
2. Order of operations
3. Use of the distributive property
4. Comparison of fractions & decimals
5. Multiplication short-cuts
6. Squaring numbers
7. Roman numerals/arabic numbers
8. Mean, median, mode
9. Sums of whole numbers

**Problems 21 - 40**
1. Addition, subtraction, multiplication & division of mixed numbers and integers
2. More multiplication short-cuts
3. Percent problems
4. Conversion problems (either way): English/metric, length, area, capacity, time
5. Consumer type problems
6. Substitution problems
7. Solving simple equations
8. Square roots/cube roots
9. Greatest common divisor (gcd) & least common multiple (lcm)
10. Number theory - prime numbers and divisors
11. Perimeter/area of: square, rectangle, circle
12. Ratio/proportion
13. Inverses

**Problems 41 - 60**
1. Sets
2. Word problems
3. Pythagorean theorem
4. Sequences
5. Volume/surface area of rectangular solid/cube
6. Base systems
7. Area of: parallelogram, rhombus, trapezoid, circle
8. Solving inequalities
9. Basic geometry facts
10. Remainder problems

**Problems 61 - 80**
1. Repeating decimals
2. More number theory
3. Powers of numbers
4. Volume of: circular cylinder, cone, sphere
5. Sequences & series
6. Multiplication of 101, 111
7. Factorial
8. Coordinate geometry
9. Probability
10. More percent type problems
11. More remainder type problems

12. More multiplication short-cuts

## HIGH SCHOOL (Source: http://www.uil.utexas.edu/academics/math_number_sense/NS_sequence_HS.pdf)

**Problems 1 - 20**

1. Addition, subtraction, multiplication, & division of Integers, Mixed Numbers, Fractions, and Decimals
2. Order of Operations
3. Use of the Distributive Property
4. Comparison of Fractions and Decimals
5. Multiplication Short-Cuts
6. Squaring Numbers
7. Conversion Problems (either way): Percent/Fractions, English/Metric, Roman Numerals/Arabic Numerals
8. Greatest Common Divisor (GCD & Least Common Multiple (LCM)
9. Percent Problems
10. Mean, Median, & Mode
11. Sums of Integers
12. Remainder Problems
13. Consumer Type Problems
14. Number Theory Problems Involving: Prime Numbers, Divisors, Sums of Divisors, etc.

**Problems 21 - 40**

1. Powers of Numbers
2. Substitution
3. Word Problems
4. Inverses
5. Absolute Value
6. Ratio/Proportion
7. Square Roots/Cube Roots
8. Sets
9. Base System Problems
10. Solving Simple Equations
11. Simultaneous Equations
12. Repeating Decimals to Fractions
13. More Remainder Type Problems
14. Perimeter & Area Problems of Polygons
15. Sequences
16. Quadratic & Cubic Equation Problems

**Problems 41 - 50**

1. Laws of Exponents
2. Right Triangle Problems
3. Coordinate Geometry Problems
4. Regular Polygon Problems
5. Inequalities

**Problems 51 - 60**

1. Applications of Theorems from Geometry
2. Direct and Inverse Variation
3. Sequences & Series (Finite & Infinite)

**Problems 51 - 70**

1. Complex Numbers
2. Logarithms & Logarithmic Equations
3. Permutations & Combinations
4. Probability
5. Conics
6. Binomial Theorem (Expansion)

**Problems 61 - 70**

1. Volume & Surface Area
2. Greatest Integer
3. Application of Remainder Theorem
4. Trigonometry
5. Determinants
6. Matrices
7. Vectors
8. Composite Functions

**Problems 71 - 80**

1. Value of Domain of a Given Function
2. Bases Involving Decimal Fractions
3. Polar/Rectangular Coordinates
4. Modular Arithmetic
5. Limits
6. Derivative
7. Slopes of Tangent Lines
8. Horizontal & Vertical Asymptotes
9. Determining Critical Values
10. Maximum & Minimum Problems
11. Definite Integration