**The Number Sense Handout**

***A Collection of Important Tricks and Concepts Important***

***To the Number Sense Test***

**Addition and Subtraction:**

The way that I tend to do addition and subtraction is by simply adding through the digits (or subtracting through the digits). Every time there is a carry-over re-train it in your brain for a second, then once you have used it you can continue to the next term.

A Standard Addition Example

Example: 19325 + 52487

1. Add the last two terms – the sum is 12, so write down the 2 and carry the 1
2. Add the tens digits plus the carry over, write down 1 and carry 1.
3. (Continue doing this, you get the idea)
4. Your final answer should be 71812.

Remember to start writing from the right side of the answer blank, and proceed towards the left side.

“Mirror Image” Subtraction with 3 Digit Numbers:

1. Subtract the units digit from the hundreds digit in the first number.
2. Multiply this number by 99.
	1. Note: Multiplying by 99 is efficiently accomplished by breaking 99 into 100 – 1. In this way you can multiply the original number by 100, then subtract the original number from this product.
3. Write down the product.
4. This trick is handy because often subtraction problems involving mirror images require lots of borrowing, so this trick eliminates that.

Example:814 – 418 = \_\_\_\_\_\_\_\_

1. 8 – 4 = 4
2. 4 \* 99 = 4 \* (100 – 1) = 400 – 4 = 396

Solution: 814 – 418 = 396.

Example: 152 – 251 = \_\_\_\_\_\_\_\_

1. 1 – 2 = -1
2. -1 \* 99 = -99

Solution: 152 – 251 = -99

One important topic that also needs to be covered is the additive inverse.

Computing the Additive Inverse

1. The additive inverse is defined as the number which, when added to the original number, gives a sum of 0. All this is fancy terminology for the fact that the additive inverse is just the negative of the number.

Example: The additive inverse of 3.152 is \_\_\_\_\_\_\_\_

1. 3.152 + (-3.152) = 0, thus the answer is -3.152

Solution: The additive inverse of 3.152 is -3.152.

Example: The additive inverse of – π is \_\_\_\_\_\_\_\_

1. Take the opposite of – π, which is just + π

Solution: The additive inverse of – π is π

**Multiplication and Division:**

Mastering the multiplication tricks is a very important part of achieving a high number sense score. The first, and most basic, trick for multiplication is often called the “rainbow trick”.

Multiplying Two 2 Digit Numbers (When there is no trick)

1. Multiply the unit’s digits. Be sure to remember the carry over if the product is greater than 9.
2. Multiply the insides and the outsides, adding the carry over. Remember the carry over from this.
3. Multiply the ten’s digits together, adding the carry over.

Example: 24 × 86 = \_\_\_\_\_\_\_\_

1. 6 × 4 = 24. Write down the 4 and carry the 2.
2. (4 × 8) + (2 × 6) + 2 = 52. Write down the 2 and carry the 5.
3. (2 × 8) + 5 = 21. Write this down.

Solution: 24 × 86 = 2124

This trick also works for multiplying a two digit by a three digit.

Multiplying a Two Digit by a Three Digit

1. Multiply unit’s digits. Remember to carry.
2. “Cover up” the hundreds digit of the three digit number and add the products of the insides and the outsides, plus the carry over.
3. “Cover up” the ones digit of the three digit number and add the products of the insides and the outsides, plus the carry over.
4. Multiply the left most digits of both numbers.

Example: 623 × 49 = \_\_\_\_\_\_\_\_

1. 3 × 9 = 27. Write down 7 and carry 2.
2. (2 × 9) + (3 × 4) + 2 = 32. Write down the 3 and carry 2.
3. (6 × 9) + (2 × 4) + 3 = 65. Write down the 5 and carry 6.
4. 6 × 4 + 6 = 30. Write down the 30.

Solution: 623 × 49 = 30537

Multiplying 2 Numbers ending with 5

1. Take a look at the tens digits.
	1. If both are even, or both are odd, write down 25.
	2. If one is even and one is odd, write down 75.
2. Add the tens digits and divide by 2. “Throw away” the decimal part of the answer and add this new number to the product of the tens digits.
3. To recap, write down the back two numbers based on the parity (whether they are even or odd) of the tens digits. Then, add the sum of the tens digits divided by two with the product of the tens digits, then take away the decimal part.

Example: 45 × 95 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. The tens digits are of different parity, so write down a 75.
2. Add the sum of the tens digits and the product of them, which is (4 + 9)/2 + (4 x 9) = 42.5 which goes to 42.

Solution: 45 × 95 = 4275

Example: 45 × 65 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. The tens digits are of the same parity, so write down 25.
2. (4 + 6)/2 + (4 x 6) = 29.

Solution: 45 × 65 = 2925

Same **Tens** Digit and Sum of **Units** Digits Is 10

1. Write down the product of the units digits. If the product is a single digit, write a 0 in front of it.
2. Write down the product of the tens digit (they are the same) and the tens digit plus one.

Example: 32 × 38 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Write down 2 x 8 = 16
2. Write down 3 x (3 + 1) = 12

Solution: 32 × 38 = 1216

Example: 74 × 76 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. 4 x 6 = 24
2. 7 x (7 + 1) = 56

Solution: 74 × 76 = 5624

Same **Units** Digit and Sum of **Tens** Digits Is 10

1. Square the unit’s digit. Make sure to write a zero in front of this square if it is a single digit.
2. Multiply the tens digits together and add the units digit to this.

Example: 42 × 62 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Multiply unit’s digits: 2 x 2 = 4. Add the zero!
2. Multiply the tens digits and add the units digit: 4 x 6 + 2 = 26.

Solution: 42 × 62 = 2604

Example: 38 × 78 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. 8 x 8 = 64
2. 3 x 7 + 8 = 29

Solution: 38 × 78 = 2964

Multiplying Two Numbers Close to 100

1. Write down the product of the differences of each number from one hundred. (Subtract each number from 100 and multiply the two differences)
2. Write down the difference of one of the numbers from the difference between the other number and 100. (That was incredibly convoluted, you’ll see in the examples)

Example: 89 × 95 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. The difference of 89 and 100 is 11. The difference of 95 and 100 is 5. The product of these two differences is 11 x 5 = 55. Write this down.
2. Taking one of the numbers, we will subtract from it the difference of the other number and 100. If we pick 89, then we need to subtract the difference of 95 and 100, or 5. This gives us 89 – 5 = 84. If we had picked 95, we would have subtracted the difference of 89 and 100, or 11. This would also be 84. (Which is no coincidence)

Solution: 89 × 95 = 8455

Example: 97 × 92 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. (100 – 97) × (100 – 92) = 24
2. 92 – (100 – 97) = 89. Remember, we could have done it the other way too.

Solution: 97 × 92 = 8924

Multiplying by 11

1. Write down the units digit at the right side of the blank
2. Going through the digits two by two, add the two digits, starting from the units digit. Continue this step until you have finished with all the digits.
3. Write down the left most number

Example: 7624 × 11 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Write down the 4
2. Now look at the last two digits. The sum of 2 and 4 is 6. Write this down.
3. Now look at 6 and 2, the next two digits. The sum is 8. Write this down.
4. Look at 7 and 6. The sum is 13 – remember to carry over this 1.
5. Finally, write down the 7, plus the carry over of 1. So write down 8.

Solution: 7624 × 11 = 83864

Multiplying by 15

1. Note that 15 is just 30 divided by 2, so multiply the number by (30/2) or divide the number by 2 and multiply by 30.

Example: 36 × 15 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. 36 divided by 2 is 18. 18 multiplied by 30 is 540.

Solution: 36 × 15 = 540

Multiplying by 25

1. 25 is essentially 100 divided by 4, so multiply the number by (100/4), or divide the number by 4 and multiply by 100.
2. If there is a remainder when you divide by four:
	1. No remainder, write down 00
	2. Remainder 1, write down 25
	3. Remainder 2, write down 50
	4. Remainder 3, write down 75

Example: 28 × 25 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. 28 divided by 4 is 7. Multiply this by 100.

Solution: 28 × 25 = 700

Example: 34 × 25 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. 34 divided by 4 is 8 with a remainder of 2. Thus, write down 8 and a 50.

Solution: 34 × 25 = 850

Multiplying by 37

1. 37 is just 111 divided by 3. Thus, divide the non-“X” number by three and multiply by 111. (The trick for multiplying by 111 is later)

Example: 12 × 37 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. 12 divided by 4 is 3. 3 multiplied by 111 is 333.

Solution: 12 × 37 = 333

Multiplying by 50

1. 50 is 100 divided by 2, so divide by 2 than multiply by 100. (Add two zeros)

Example: 57 × 50 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. 57 divided by 2 is 28 with a remainder 1. This remainder means there is an extra 50, so the answer is 2850.

Solution: 57 × 50 = 2850

Multiplying by 75

1. 75 is 300 divided by 4, so divide by 4 than multiply by 300.

Example: 75 × 48 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. 48 divided by 4 is 12. Multiplying 12 by 300 is simply 3600.

Solution: 75 × 48 = 3600

Multiplying by 99

1. 99 is (100 – 1), so multiply the number by 100 and subtract the number from the product.

Example: 48 × 99 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. 48 x 99 = 48(100 – 1) = 4800 – 48 = 4752

Solution: 48 x 99 = 4752

Multiplying by 101

1. If the number has 1 digit, write it down twice with a zero in the middle. If the number has 2 digits, write the number down twice. If the number has 3 digits, write down the ones and tens digits, then the hundreds plus the ones digits, then the tens digit, then the hundreds digit.
2. Another way to think about multiplying 101 by a three digit is:

 101

×ABC

 ABC

 ABC 00

Example: 101 × 382 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Write down 82, then 2 + 3 = 5, then 38.

Solution: 101 × 382 = 38582

Multiplying by 111

1. Write down the units digit
2. Add the tens and units digits together. Make sure to carry
3. Add in groups of 3 from right to left (similarly as the 11 trick). Continue this process.
4. Add the 2 left digits.
5. Write down the leftmost digit

Example: 8357 × 111 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Write down the 7
2. 5 + 7 = 12. Write down 2, carry the 1.
3. 3 + 5 + 7 + (1) = 16. Write down 6, carry the 1.
4. 8 + 3 + 5 + (1) = 17. Write down 7, carry the 1.
5. 8 + 3 + (1) = 12. Write down 2, carry the 1.
6. 8 + (1) = 9. Write this down.

Solution: 8357 × 111 = 927627

Multiplying by 125

1. Note that 125 is 1000 divided by 8. Thus, divide the number by 8 and add three zeros.

Example: 96 × 125 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. 96 divided by 8 is 12. Write down 12000.

Solution: 96 × 125 = 1200

Multiplying by 375

1. Note 375 = 3000/8. Thus you divide by 8 and multiply by 3000.

Example: 440 × 375 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. 440 divided by 8 is 55. 55 x 3000 = 165000.

Solution: 440 × 375 = 165000

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Multiplying by 429

1. Note 429 = 1001 x (3/7). Thus you can multiply the number by 3/7, then by 1001.

Example: 84 × 429 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. 84 times 3/7 is (84/7) = 12 then 12 x 3 = 36. 36 multiplied by 1001 is 36036.

Solution: 84 × 429 = 36036

Multiplying by 3367

1. Note that 3367 x 3 = 10101. Thus, find how many times 3 goes into the other number.
2. Write this result repeated 3 times, making sure to insert a placeholder zero if needed.

Example: 12 × 3367 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. 12 divided by 3 is 4.
2. Writing 4 three times, inserting placeholders, makes 40404.

Solution: 12 × 3367 = 40404

Example: 63 × 3367 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. 63 divided by 3 is 21.
2. 21 written three times is 212121

Solution: 63 × 3367 = 212121

**Fractions:**

Before you learn the tricks, make sure that you know how to add, subtract, multiply, and divide fractions. The basic knowledge is always necessary, so that you can fall back to it as a default or better understand the tricks.

Adding Two Fractions When the Denominators Do Note Share Any Factors

Note that this property actually applies to many questions.

1. “Cross multiply” – multiply the denominator of the first number by the numerator of the second number, and multiply the numerator of the first number by the denominator of the second number, then add your products. Put this in the numerator of your answer.
2. The denominator of your answer will be the product of the two denominators.

Example: 3/7 + 2/3 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. The first step is to add 3 x 3 +7 x 2 = 23. Write this as the numerator.
2. Now 7 x 3 = 21. Write this as the denominator.

Solution: 3/7 + 2/3 = 23/21

See pretty cool, right? This trick can actually be generalized into a very important mathematical property. (So memorize this)

When adding a/b + c/d, and when b and d do not share any factors (they are relatively prime), we have:

$$\frac{a}{b}+\frac{c}{d}= \frac{ad+bc}{bd}$$

Think about just why this is true for a while.

Adding Reciprocals

Remember that a reciprocal is essentially a number “flipped over”.

1. Square the difference of the numerators
2. Multiply the denominators
3. Place the result of number 1 over number 2. Make it a mixed number if necessary. Write this down
4. Write down 2, plus any carry over if there is.

Example: 8/9 + 9/8 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. (8 - 9)2 = 1
2. 9 x 8 = 72
3. Write down 1/72
4. Write down 2

Solution: 8/9 + 9/8 = 2 ½

Multiplying A Whole Number by A Fraction

This method will only work if the whole number is the same as the numerator of the fraction.

1. Subtract the numerator from the denominator and square this
2. Place this value on top of the denominator
3. If the numerator was greater than the denominator, add the difference of the numerator and the denominator to the whole number portion and write this down. If it was the other way around, then subtract the difference from the whole number portion.

Example: 8 x 8/3 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Mixed Number

1. (8 – 3)2 = 25
2. 25/3 = 8 1/3 Remember that we have to break it into a mixed number.
3. Add the difference of the numerator and the denominator to the numerator: 8 + 5 = 13

Solution: 8 x 8/3 = 21 1/3

Multiplying Mixed Numbers With Equivalent Fractional Parts

1. Add the whole number parts and multiply this by the fraction.
2. Multiply the whole number parts – add this to the result in step 1.
3. Multiply the fractional parts (square it)

Example: 2 ¾ x 6 ¾ = \_\_\_\_\_\_\_\_\_\_\_\_\_ Mixed Number

1. (2 + 6) x ¾ = 6
2. 2 x 6 = 12. 12 + 6 = 18. Write this down.
3. ¾ x ¾ = 9/16

Solution: 2 ¾ x 6 ¾ = 18 9/16